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## C.U.SHAH UNIVERSITY

Summer-2015

## Subject Code: 4SC02MTC1 Subject Name: Mathematics-II

Course Name: B.Sc. (Pure)
Semester:II

Date: 18/5/2015
Marks:70
Time:10:30 TO 01:30

## Instructions:

1) Attempt all Questions in same answer book/Supplementary.
2) Use of Programmable calculator \& any other electronic instrument prohibited.
3) Instructions written on main answer book are strictly to be obeyed.
4) Draw neat diagrams \& figures (if necessary) at right places.
5) Assume suitable \& perfect data if needed.

Que. 1 Attempt the following
Find the equation of cone having vertex $(0,0,0)$ and passing through the curve
1 $x^{2}+y^{2}=4, z=2$.
2 Find the real and imaginary part of $\exp \left((5+3 i)^{2}\right)$.
3 Find the magnitude and argument of $(\sqrt{3}+i)$.
4 Check the convergence of a sequence $(-1)^{n}$.
5 Find the order and degree of an differential equation $\left[1+\frac{d y}{d x}\right]^{5}=\sqrt{1+\frac{d^{2} y}{d x^{2}}}$.
6 Solve $D^{2} y-y=0$.
7 Evaluate $\int_{0}^{\pi / 2} \sin ^{7} x \cos ^{5} x d x$.
Que. 2 Attempt the following:
Find the equation of tangent plane at a point $(1,0,-1)$ on the cone
1
$2 x^{2}+y^{2}-2 z^{2}=0$.
2 If $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represents one of a set of three mutually perpendicular generators of the cone $5 y z-8 x z-3 x y=0$,find the equations of other two.
3 Prove that the cone $a x^{2}+b y^{2}+c z^{2}=0$ and $\frac{x^{2}}{a}+\frac{y^{2}}{b}+\frac{z^{2}}{c}=0$ are reciprocal of each other.


Que. 3 Attempt the following:
1 Solve $\left(D^{2}-3 D+2\right) y=e^{-2 x}+\sin 3 x+3$.
2 Solve $y^{\prime \prime}+y=e^{x} \cos 2 x$.
$3 \operatorname{Solve}\left(D^{3}-D^{2}-D+1\right) y=1$.
Que. 4 Attempt the following:
1 Solve $\left(x^{2} D^{2}-3 x D+4\right) y=x^{2}$.
2 Solve $x^{2} y^{\prime \prime}-x y^{\prime}+y=\log x$.
3 Solve $(x-3)^{2} \frac{d^{2} y}{d x^{2}}+3(x-3) \frac{d y}{d x}-8 y=0$.
Que. 5 Attempt the following:
1 State and prove the D'Moivre's theorem.
2 Express $\cos 6 \theta$ in terms $\cos \theta$.
3 Find all the values of $(1+i)^{1 / 3}$.
Que. 6 Attempt the following:
If $x+i y=\cosh (u+i v)$, prove that (a) $\frac{x^{2}}{\cosh ^{2} u}+\frac{y^{2}}{\sinh ^{2} u}=1$ (b)
1

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\frac{x^{2}}{\cos ^{2} v}-\frac{y^{2}}{\sin ^{2} v}=1 .
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2 Prove that $(\sqrt{i})^{\sqrt{i}}=e^{-\alpha} \operatorname{cis}(\alpha)$ where $\alpha=\frac{\pi}{4 \sqrt{2}}$.
3 Prove the following results:
(a) $\left[\sin (\alpha+\theta)-e^{i \alpha} \sin \theta\right]^{n}=e^{-i n \theta} \sin ^{n} \alpha$
(b) $\sin (\alpha-n \theta)+e^{-i \alpha} \sin n \theta=e^{-i n \theta} \sin \alpha$.

Que. 7 Attempt the following
1 Every convergent sequence has unique limit point.
2 Check the convergence of the following sequences:
(a) $\left\{\frac{1}{2^{n}}+\frac{1}{3^{2 n}}\right\}$
(b) $\left\{\frac{n^{2}+3 n+5}{2 n^{2}+5 n+7}\right\}$.

3 Derive reduction formula for $\int \cos ^{n} x d x$.
Que. 8 Attempt the following
1 Evaluate $\int \sin ^{4} x d x$.
2 Evaluate $\int \sin ^{4} x \cos ^{2} x d x$.
3 Evaluate $\int_{0}^{1} x \sin ^{-1} x d x$.


